International Journal of Mathematics and Physical Sciences Research ISSN 2348-5736 (Online)
Vol. 4, Issue 1, pp: (15-25), Month: April 2016 - September 2016, Available at: www.researchpublish.com

# Measuring the Performance of Autoregressive Integrated Moving Average and Vector Autoregressive Models in Forecasting Inflation Rate in Rwanda

[1]Joselyne INGABIRE, [2]Dr. Joseph K. Mung'atu.

[1,2] Faculty of applied sciences, Department of statistics and actuarial sciences, Jomo Kenyatta University of Agriculture and Technology, Kigali, Rwanda

***Abstract:*** **The aim of this study is to test and distinguish which of ARIMA and VAR models performs best in forecasting inflation in Rwanda. In order to fulfil this objective observed quarterly data from 2000Q1 to 2015Q1 on economic variables such as the Consumer Price Index, Money Supply, Gross Domestic Product, T-bills Rate and Exchange Rate are used to build models. Box-Jenkins approach is used to build ARIMA model on Consumer Price Index data and it has been found that ARIMA (3, 1, 4) perform better. In VAR analysis Johansen's Cointegration approach is applied and the results showed that there is long run and short run relationship between dependent variable (inflation) and independent variables. The performance of these models in forecasting inflation is evaluated using Root Mean Squared Errors (RMSE), Mean Absolute Errors (MAE), Mean Absolute Percentage Errors (MAPE) and the results showed that ARIMA (3, 1, 4) perform better than VAR model in forecasting inflation in Rwanda.**

***Keywords:*** **Inflation, forecast, Cointegration, ARIMA and VAR Models.**

## 1. INTRODUCTION

High price constitutes the major economic problems in emerging market economies as it affects country's competitiveness, financial situation of households and companies, it leads to uncertainty making domestic and foreign investors reluctant to invest in the economy while stable prices allow market participants, both domestic and foreign, to make informed decisions and adjust their decisions about spending, saving and investing in welfare.

Rwanda experienced the problem of inflation volatility , for instance during the 2001-2005 periods( Musoni J . Rutayisire , 2008) inflation was kept at an average rate of 6.7% but during 2006 the rate change 0.6% ; this was due to first of all the increase in salaries in 2006 and 2008 the oil shocks. The annual average underlying inflation rate is 3.8 % in September 2013 from the previous month 3.7 % and the headline inflation has eased over the last eight months of 2014, falling from 2.38% in January to 0.90% in August and averaging around 2.3% for the entire 8 months period against 4.0% of the same period in 2013. This continued decline in inflation was due to the significant drop in food inflation from 2.52% in January to 0.20% in August 2014 with an average of around 3.0% in the first eight months of 2014 against 4.8% of the same period in 2013 (Gerardi & Li, 2010). To prevent this volatility in inflation, policymakers used econometrics methods which use time series analysis and the main objective of time series analysis is to forecast future values of macroeconomic variables by studying the behavior of data in the past. Building economic indicators in short-term and long term period helps in more accurate prediction of the current and next period, these are crucial steps for decision makers, which rely on these projections for analyzing and planning the economy in the long term. Many studies was conducted on inflation in Rwanda for instance, the study conducted by(UFITINEMA.R,2010.) on factors affecting inflation in Rwanda using regression model to analyze the extent to which changes in Money Supply; in Output (Real GDP); in Exchange rate and interest rate influence the inflation used the data from 1990-2009, the study found that an increase in Money supply has a positive and significant influence on Inflation. Contrary to Real GDP have a negative and significant influence on Inflation.

(KAYISIRE Pascal,2014 ) developed Philips curve models in forecasting inflation in Rwanda using CPI ,money supply ,output gap exchange rate. He found that the Philips curve and augmented Philips curve forecasts outperform the AR benchmark forecasts at one and two quarter horizons. The output gap, exchange rate and money supply are found to be good predictors of inflation in Rwanda in the generalized Philips curve context.

Through there is substantial amount of research on inflation in Rwanda, less attention has been given to predicting inflation by comparing different models, In view of this, our study intends to forecast inflation in Rwanda by making a comparison of forecast performance of VAR and ARIMA models, this can make an improvement in forecasting inflation and help the policymakers to make a future prediction of the economy.

## 2. LITERATURE REVIEW

In time series analysis, The ARIMA model is due to Box and Jenkins (1970) work for short term forecasting of a large variety of time series data. It is a univariate time series model which underlying the assumption that the time series to be forecasted has been generated by a stochastic process, It is assumed that past values of the series plus previous error terms contain information for the purposes of forecast (Koopman & Commandeur, 1994)The main advantage of ARIMA forecasting is that it requires data on the time series in question only and ARIMA models frequently outperform more sophisticated structural models in terms of short-run forecasting ability; ARIMA models are flexible and widely used (Casella, Fienberg, & Olkin, 2006)

In multivariate analysis there are different methods used in modeling and forecast inflation; one of them is a Vector autoregressive (VAR) model which was introduced by Sims (1980) as a technique that could be used by macroeconomists to characterize the joint dynamic behavior of a collection of variables without requiring strong restrictions of the kind needed to identify underlying structural parameters (Zivot & Wang, 2006).

(Johansen, 1995) and many other authors suggest that for the calculation of forecast of economic indicators VAR models should be applied because all variables in these models are endogenous and therefore not a single variable may be removed when explanations for the behavior of other variables are offered. Different studies used ARIMA and VAR models in understanding the dynamic movement in macroeconomic variables especially in inflation.

Adopting a VAR model to study inflation in Ghana, (Ocran, 2003) identified exchange rate, foreign prices and Terms of Trade as determinants of inflation in the long-run. This study hence failed to find evidence for the influence of excess money supply on inflation (in the long-run). In the short-run, inflation inertia, exchange rate, money growth and Treasury bill rate were found to be important determinants of inflation.

(Meyler, 2015) empirically developed univariate autoregressive moving average model in forecasting Irish inflation using harmonized index of consumer prices quarterly data form 1976 Q1 to 1998Q4, in their study ARIMA models are theoretically justified and can be surprising robust with respect to multivariate approaches, based on their results they concluded that multivariate models generally perform better than ARIMA models over long time horizon.

In forecasting Suiss inflation using VAR models(Lack,2006.) used different macroeconomic variables which are mortgage loans, M2, M3, the bond yield and index for rent and GDP, by using simulation; the study showed that by combining forecast from different VAR models a significant reduction in the root mean squared error can be obtained and VAR analysis indicated that Bank loans and Monetary aggregate M3 are the most important variables for inflation forecast.

In predicting inflation in Ghana by comparing cointegration and ARIMA models (Alnaa & Abdul-Mumuni, 2005) based on Consumer Price Index, Money Supply, Interest Rate and Exchange Rate, in determine which model is more efficient than the other based on the RMSE, they found that VAR model has the lowest value of RMSE, Thus the VAR model can be associated with much more stability in inflation if used for prediction.,

The study conducting by (Gathingi, 2014)on modeling inflation in Kenya using ARIMA and VAR models used the data on money supply, murban oil price, exchange rate and CPI; In comparing the forecast performance of these two modes using RMSE, MAE and MAPE, the study found that VAR model is the best model in forecasting inflation in Kenya since it has the smallest errors than ARIMA model.

## 3. RESEARCH METHODOLOGY

In this section methods for data analysis is discussed those are unit root test for testing the stationarity followed by Box-Jenkins approach for ARIMA model and lastly in multivariate case Vector Autoregressive models is discussed where Johansen's cointegration technique is applied in the study.

### 3.1. Stationarity of time series data:

The first step in time series analysis is to check if the data are stationary because when not stationary data are used, the results leas to spurious results this gives a false relationship between the variables. A time series is said to be stationary if there is no systematic change in mean, and change in variance and if strictly periodic variations have been removed. If a time series is stationary, then any shocks that occur are transitory, their individual effects decay and eventually disappear as the time increase whereas if a series is not stationary, then shocks have permanent effect on the series(Gospodinov, Marı, & Pesavento, 2013) ,different methods are used to test if the data are stationary but in this study Augmented Dickey Fuller test was conducted to test stationarity.

The Augmented Dickey-Fuller (ADF) test is developed by Dickey and Fuller (1979, and fuller 1976), It relies on the assumption that the residuals are independent and identically distributed, ADF is used to test the presence of unit roots (Burke & Hunter, 2005). A random walk with drift and trend is given by:

$$\Delta y_t = \mu + \lambda t + \Psi y_{t-1} + \sum_{i=1}^{p} \alpha_i \Delta y_{t-i} + u_t$$

If $\mu$ and $\lambda$ are all zero it corresponds to modeling a random walk and if $\lambda = 0$ corresponds to modeling random walk with a drift, where $y_t$ is the variable under consideration at time period t and $\Delta y_t = y_t - y_{t-1} + u_t, \mu$ is the drift term, $\lambda$ is linear trend term and $u_t$ is the error term. If the data are not stationary, we must transform them using differentiation by starting from the first difference which is the change from one period to the next. If after first difference the series is still not stationary the second differentiation is need and the third until the series became stationary as the model fitting is carried only on stationary series.

### 3.2. Autoregressive integrated moving average model:

Autoregressive integrated moving average (ARIMA) model was first popularized by Box and Jenkins (1970). It is for analysis of univariate time series where a variable is explained by its previous values and its error terms. The ARMA model is a combination of two univariate time series models which are Autoregressive (AR) model and Moving Average (MA) models and the ARIMA model is applied in the case where the series is non-stationary; an initial differencing step can make ARMA model applicable to an integrated stationary process(Wei,1994) .

The ARIMA model within its order is presented as ARIMA (*p*, *d*, *q)* model where *p* refers to the number of autoregressive lags, *d* refers to the order of integration that makes the data stationary and *q* gives the number of moving average lags ,(Casella et al., 2006) mathematically these models are given by:

An Autoregressive model $y_t = \mu + \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \cdots + \varphi_p y_{t-p} + u_t,$

Where $y_t$ is the stationary dependent variable being forecasted at time t, $y_{t-1}, y_{t-2}, \ldots, y_{t-p}$ is the response variable at time lags t-1,t-2,…,t-p respectively, $u_t$ is the error term at time t with mean zero and a constant variance,

$\mu, \varphi_1, \varphi_2, \ldots . \varphi_p$ are the parameters to be estimated.

A moving average model: $y_t = \mu + u_t + \theta_1 u_{t-1} + \theta_2 u_{t-1} + \cdots + \theta_q u_{t-p}$

Where : $u_t(t = 1,2, \ldots . . q)$ is white noise process with $E(u_t)$=0 and $Var(u_t) = \sigma^2$, q is the number of lags in the moving average and $\theta_1, \theta_2, \ldots \theta_q$ are parameters to be estimated.

ARMA (p, q) is given by:

$$y_t = \mu + \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \cdots + \varphi_p y_{t-p} + \theta_1 y_{t-1} + \theta_2 y_{t-2} + \cdots + \theta_q y_{t-q} + u_t$$

$$y_t = \mu + \sum_{t=1}^{p} \varphi_p y_{t-p} + \sum_{t=1}^{q} \theta_q y_{t-q} + u_t$$

As mainly the economic time series are non –stationary in the nature afterward and the differentiation is apply, in that case ARIMA model is used, in the general form ARIMA (p, d, q) is written as:

$$\varphi_p(B)(1-B)^d Y_t = \mu + \theta_q(B)u_t$$

Where $\varphi_p$ is the Autoregressive operator of order p, $\theta_q$ is the Moving Average operator of order q, $\mu$ is the constant term and $u_t$ is the errors term at time t. As mentioned above Box-Jenkins approach is followed in building ARIMA model and it is carried out in four steps which are identification, estimation, diagnostic checking and forecasting(Mccullough, 2005)

### 3.3. Vector autoregressive model:

The Vector Autoregressive (VAR) model, proposed by Sims (1980), is one of the most successful, flexible and easy to use for analysis of multivariate time series. It is applied to grasp the mutual influence among multiple time series. VAR models extend the univariate autoregressive (AR) model to dynamic multivariate time series by allowing for more than one evolving variables, all variables in a VAR model are treated symmetrically in a structural sense; each variable has an equation explaining its evolution based on its own lags and the lags of the other variables in the model (Bash, 2015)

In its basic form, a VAR model consists of a set of K endogenous variables,

Let $y_t = (y_{1t}, y_{2t}, y_{3t,\dots,} y_{kt})$' for k = 1, 2…K.

The VAR model with p lags can be expressed as:

$$y_t = \mu + A_1 y_{t-1} + A_2 y_{t-2} + \cdots + A_p y_{t-p} + \epsilon_t$$

Where $y_t$ *is* (K × 1) matrix of variables, $A_i$ are ($K \times K$) coefficient matrices for $i$= 1… $p$ and $\epsilon_t$ is ($K$ × 1) unobservable zero mean white noise with E ( $\epsilon_t$) = 0 and time invariant positive definite covariance matrix. E ( $\epsilon_t, \epsilon_t^T$) =$\sum \epsilon$ and $\mu$ is an ($K$ × 1) vector of constants (intercept).

Estimates of $A_i$ contain information on short run adjustments while $\mu$ contain information on long run adjustments changes in $y_t$(Zivot & Wang, 2006) in the case where the variables in VAR system are cointegrated , we use Vector Error Correction Model(VECM) which is represented as follows:

$$\Delta y_t = \mu + \Pi y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-1} + \varepsilon_t$$

Where: $\Pi = \sum_{i-1}^{p} A_i - I$ and $\Gamma_i = -\sum_{j=i+1}^{p} A_j$

The matrix $\Pi$ is called the long-run impact matrix and $\Gamma_i$ is the short run impact matrices. In the VECM model, $\Delta y_t$ and its lags are I (0), the term $\Pi y_{t-1}$ is the only one which includes potential I (1) variables and for $\Delta y_t$ to be I (0) it must be the case that $\Pi y_{t-1}$ is also I (0); Therefore, $\Pi y_{t-1}$ must contain the cointegrating relations if they exist.

In testing if cointegration in variables exist, Johansen approach is used which based on trace and Maximum Eigen value tests those are given by:

$$\lambda_{trace} = -T \sum_{i=r+1}^{n} ln\,(1-\hat{\lambda}_i)$$

$$\lambda_{max} = -T ln(1-\hat{\lambda}_{r+1})$$

Here $T$ is the sample size and $\hat{\lambda}_i$ is the eigenvalues. The trace test tests the null hypothesis of $r$ cointegrating vectors against the alternative hypothesis of $n$ cointegrating vectors whereas the maximum eigenvalue test tests the null hypothesis of $r$ cointegrating vectors against the alternative hypothesis of $r$ +1 cointegrating vectors (Burke & Hunter, 2005).

The performance evaluation of ARIMA and VAR models in forecasting inflation in Rwanda is evaluated by using Mean Absolute Percentage Errors (MAPE), Mean Absolute Errors (MAE) and Root Mean Squared Errors (RMSE), the best model is the one which present the smallest errors (Makridakis & Hibon, 2013)

Mathematically these measures are given by:

$$RMSE = \sqrt{\frac{\sum_{t=1}^{T}(y_t - \hat{y}_t)^2}{T}},\ MAE = \frac{\sum_{t=1}^{T}|y_t - \hat{y}_t|}{T},\ MAPE = \left[\!\left[\frac{1}{T}\sum_{t=1}^{T}\left|\frac{(\hat{y}_t - y_t)}{y_t}\right|\right]\!\right] * 100\%$$

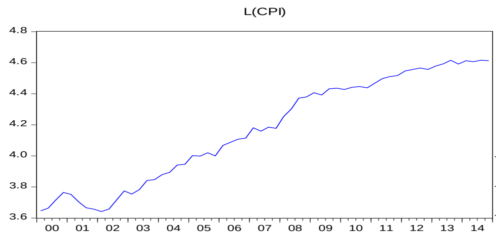
Where $y_t$ and $\hat{y}_t$ , are the actual and the forecast values of the dependent variable and T is the forecast sample size (Thomas, 2009), the advantage of using MAPA is that it relative measure express errors as percentage of the actual on other side RMSE measure the uncertainty in forecasting whereas MAE influenced less by errors (Kollo & Von Rosen, 2005).

## 4. DATA ANALYSIS AND RESULTS

This part shows the analysis of the test results for both the ARIMA and VAR models, secondary quarterly data on macroeconomic variables which are Consumer Price Index (CPI), Money supply (M3), Gross Domestic Product (GDP), Treasury Bills Rate (TBR) and Nominal Exchange Rate (ER) from 2000Q1 to 2015Q1 collected by National Bank of Rwanda was used and in building models we have used data from 2000Q1 to 2013Q4 while left data were used in evaluating the forecasting performance of models.

### 4.1. Testing stationarity:

Fig 1-10 below show the graphs in log levels and in log first difference on CPI, M3, NER, NGDP and TBR. In log levels all variables have positive and increasing trend except the TBR which represent some variation and the increasing trend, this implies that all variables are not stationary at level. On the other hand in log first difference all variables have no trend so they became stationary after first difference.



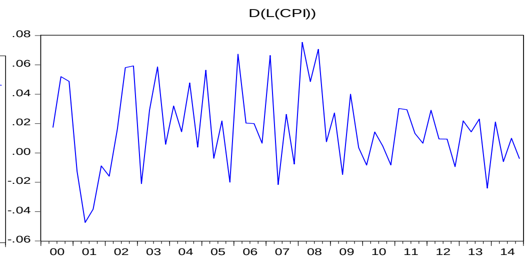
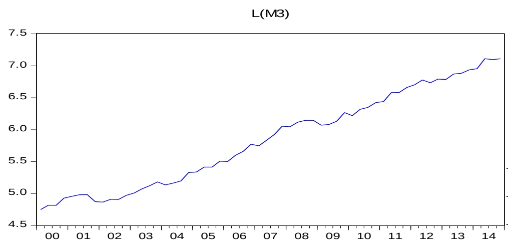




**Fig 1. Time plot of LCPI at level** **Fig 2. Time plot of LCPI at first difference**



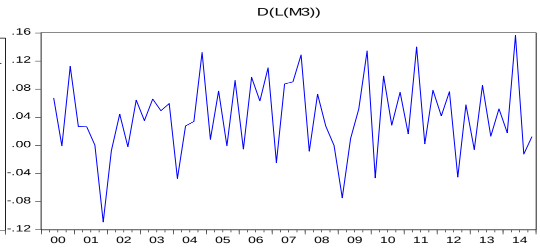
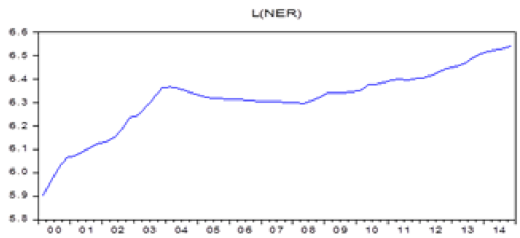




**Fig 3. Time plot of M3 at level** **Fig 4. Time plot of M3 at first difference**



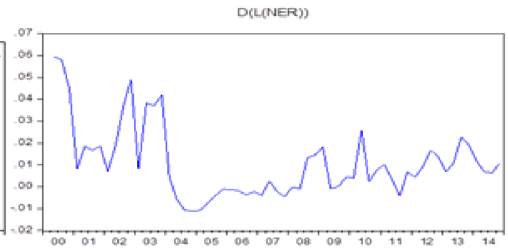
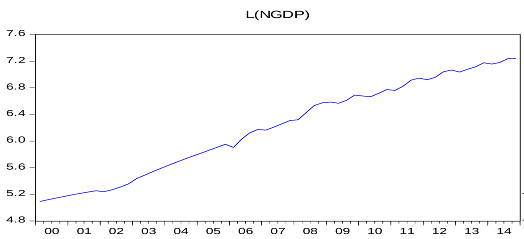




**Fig 5. Time plot of NER at level** **Fig 6. Time plot of NER at first difference**



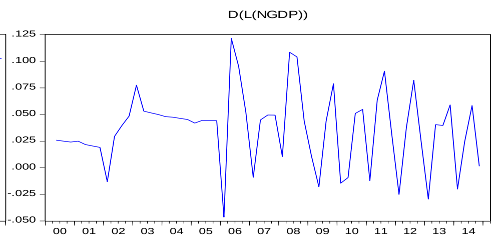
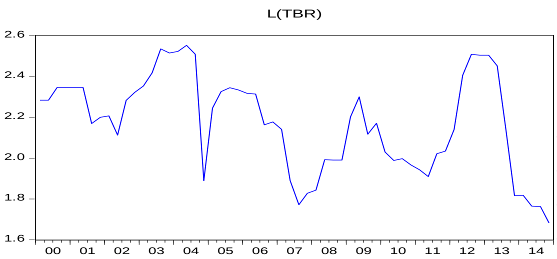




**Fig 7. Time plot of NGDP at level** **Fig 8. Time plot of NGDP at first difference**



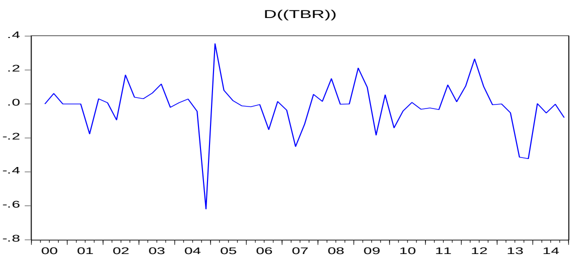




**Fig 9. Time plot of TBR at level** **Fig 10. Time plot of TBR at first difference**

The stationarity of all variables after first difference is confirmed by ADF unit root test ,based on TABLE I below, the null hypothesis of the presence of unit root is not rejected for all variables since their P-values are greater than 5% level of significance, this implies that all variables are not stationary at level whereas the null hypothesis of not stationary series at first difference is rejected as all P-values are less than 5% level of significance, this show the evident that all series are not stationary at level but they are stationary at first difference.

**TABLE I. results of ADF unit root test on all variables**

| variables | At level | | | | At first difference | | | |
|---|---|---|---|---|---|---|---|---|
| | t-stat | Critical values | | Prob | t- stat | Critical values | | Prob |
| | | 5% | 10% | | | 5% | 10% | |
| CPI | -1.345 | -3.489 | -3.173 | 0.866 | -7.271 | -3.489 | -3.173 | 0.000 |
| M3 | -2.558 | -3.487 | -3.172 | 0.300 | -9.428 | -3.489 | -3.173 | 0.000 |
| NER | -2.411 | -3.489 | -3.173 | 0.370 | -3.888 | -3.489 | -3.173 | 0.018 |
| GDP | -0.790 | -3.173 | -3.490 | 0.960 | -7.492 | -3.490 | -3.173 | 0.000 |
| TBR | -2.471 | -3.487 | -3.172 | 0.340 | -7.318 | -3.489 | -3.173 | 0.000 |

### 4.2. ARIMA Model:

#### 1. Identification:

We build the ARIMA (p, d, q) model on CPI series only by determine the order of Autoregressive process(p),order of integration(d) and the order of Moving Average (q). Since CPI became stationary after first difference the integrated order is 1. The ACF and the PACF of differenced LCPI are used to identify the order of p and q, the results indicated that ACF and PACF of differentiated CPI are dying out, so we have both the Autoregressive process and the Moving average process and the candidate models are: ARIMA(1,1,1),ARIMA(1,1,2), ARIMA (2,1,2), ARIMA(2,1,3), (3,1,4) and ARIMA(4,1,2).The good model from this list is the one which is parsimonious and minimize Akaike Information Criteria (AIC) (Johansen, 1995) Standard Error (SE) and maximize Log Likelihood Ratio (LR); as shown in TABLE II below, ARIMA (3, 1, 4) has minimum values compared to the other models so in this study we select ARIMA (3, 1, 4) as the good fitted model in the univariate case.

**TABLE II. ARIMA model selection**

| Model | AIC | LR | SE |
|---|---|---|---|
| ARIMA (1,1,1) | -4.156 | 115.21 | 0.2900 |
| ARIMA (1,1,2) | -4.385 | 122.39 | 0.0260 |
| ARIMA(2,1,2) | -4.260 | 117.93 | 0.0270 |
| ARIMA(2,1,3) | -4.326 | 117.48 | 0.0265 |
| ARIMA(3,1,4)* | -4.53* | 125.68* | 0.0234* |
| ARIMA(4,1,2) | -4.34 | 117.85 | 0.0258 |

*Best selected model

#### 2. Estimation and model checking:

The selected ARIMA (3, 1,4) model is estimated using Ordinary Least Squared and the results are shown in TABLEIII below

**TABLE III. The results of estimated ARIMA (3, 1, 4) model**

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|---|---|---|---|---|
| C | 0.017696 | 0.004819 | 3.672176 | 0.0006 |
| AR(1) | 0.238958 | 0.113365 | 2.107876 | 0.0408 |
| AR(2) | 0.389761 | 0.105021 | 3.711277 | 0.0006 |
| AR(3) | -0.679233 | 0.093602 | -7.256604 | 0.0000 |
| MA(1) | -0.112366 | 0.145001 | -0.774931 | 0.4425 |
| MA(2) | -0.351023 | 0.125933 | -2.787376 | 0.0078 |
| MA(3) | 0.491298 | 0.121864 | 4.031520 | 0.0002 |
| MA(4) | 0.525130 | 0.134471 | 3.905160 | 0.0003 |

The estimated equation is written as:

$$CPI_t = 0.02 + 0.24CPI_{t-1} + 0.39CPI_{t-2} - 0.68CPI_{t-3} - 0.11y_{t-1} - 0.35u_{t-2} - 0.49u_{t-3} + 0.53u_{t-4}$$

Based on Box-Jenkins approach, the diagnostic stage deals with analyzing the behavior of residuals and coefficients after estimation and the good model is the one with significant coefficients and white noises residuals.

The results of estimated ARIMA (3, 1, 4) model in TABLE III above show that all parameters are significantly different from zero as their P-values are less than 5% level of significance. The ACF and PACF of residuals and the Ljung Box test are used to test the autocorrelation of residuals , Jarque-Bera test and Q-Q plot on the residuals are used to test the normality lastly white's test is applied for heteroscedasticity test(Mahdi, 2011).As shown in the TABLEIV below, the results from Ljung-Box Q-statistic test indicates the absence of serial correlation in residuals as the Q- statistic and their corresponding P-values at different lags are bigger than 5% level of significance; based on Jarque-Bera test, the errors are almost normally distributed as the corresponding probability is 0.38 which is greater than 5% level of significance and white test showed that errors are homoscedastic since the F- statistic is 0.96 and its corresponding P–value is 0.58 which is greater than 5% level of significance.

**TABLEIV. Model checking for ARIMA (3, 1, 4)**

| Test | | Statistic | | Prob |
|---|---|---|---|---|
| Ljung –Box test | | Q-stat= 7.05 | | 0.22 |
| Jarque-Bera test | | 1.93 | | 0.38 |
| White test | | F- stat=0.96 | | 0.58 |

By observing the histogram in Fig11 and Q- Q plot on residuals in Fig12 we can conclude that the errors are almost normally distributed.

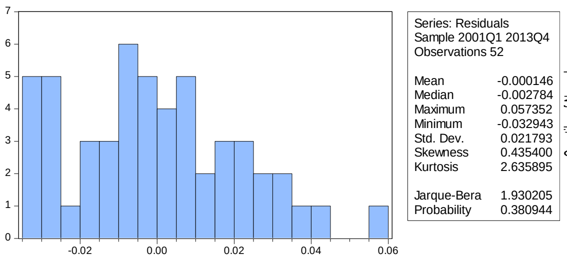



**Fig 11. Histogram of residuals**

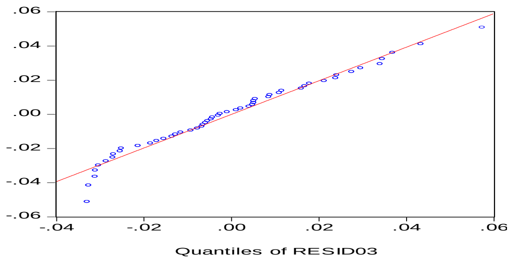



**Fig 12. Q-Q plot of residuals**

## 4.3. VECTOR AUTOREGRESSIVE MODEL:

In the Vector Autoregressive model, CPI is the dependent variable while Money supply (M3), Gross Domestic Product (GDP), Treasury bills rate (TBR) and Exchange rate (NEX) are independent variables. We estimate a VAR model with optimum lag length with a constant and trend, lag 2 was chosen based on the lag order selection criteria which are LR, FPE and AIC (Helmut, 1980) and the residuals test on selected model showed that multivariate errors are white noise. As all variables are stationary after first difference, we have to test if there is a long run relationship between the considered variables using Johansen's cointegration technique and the results as shown in TABLE V indicated that the null hypothesis of no cointegration is reject for both trace and Max- Eigen statistics as their statistics are greater than critical values and the corresponding probabilities are lower than 5% level of significance. On the other hand the null hypothesis of one cointegrating equation is not rejected for both trace and Max –Eigen value tests as their test statistics are less than critical values at 5% level of significance. This indicates that there exist one cointegrating equation and this implies that there is a long run relationship between inflation (dependent variable) and the independent variables.

**TABLE V. Results of cointegration test**

| HypothesizedN0. of CE(S) | Eigen value | Trace test | | | Maximum Eigenvalue test | | |
|---|---|---|---|---|---|---|---|
| | | Trace-stat | Cr v 5% | Prob | Max-Eigen stat | Cr v 5% | Prob |
| None* | 0.483 | 70.950 | 69.819 | 0.041 | 34.91595 | 33.877 | 0.037 |
| At most 1 | 0.322 | 36.034 | 47.856 | 0.395 | 20.60467 | 27.584 | 0.300 |
| At most 2 | 0.165 | 15.429 | 29.797 | 0.752 | 9.537891 | 21.132 | 0.786 |
| At most 3 | 0.102 | 5.8916 | 15.495 | 0.708 | 5.675874 | 14.265 | 0.655 |
| At most 4 | 0.004 | 0.2157 | 3.841 | 0.642 | 0.215749 | 3.841 | 0.642 |

The long run relationship where LCPI is the dependent variable is given by the following equation:

$$LCPI(-1) = -3.99 + 0.26LM3(-1) + 0.60NER(-1) - 0.87LGDP(-1) - 0.065TBR(-1)$$

| | | | | |
|---|---|---|---|---|
| SE | (0.0549) | (0.0857) | (0.0639) | (0.0253) |
| **T-stat** | **[4.8101]** | **[7.0068]** | **[-13.5462]** | **[-2.5709]** |

Based on the above equation the coefficients of the variables are long run coefficients, the results indicated that there is a positive relationship between Money aggregate and inflation this indicates that money aggregate and inflation are directly proportional, increase in money supply create a high inflation rate while a reduction in money supply leads to lower inflation. It is found that increase in exchange rate increase inflation whereas a decrease in exchange rate reduce inflation.

On other hand there is a negative relationship between Gross Domestic Product GDP and inflation this implies that when there is a high grow rate inflation is low while a reduction in Growth domestic Product produce a high inflation rate. Lastly Treasury Bills Rate exhibits negative relationship with inflation thus an increase in treasury bills rate leads to lower inflation rate while a decrease in treasury bills rate produce high inflation rate.

Since we have long run relationship we proceed to test if there is short run dynamic between variables where inflation is dependent variable and we estimate VECM with 2 lags as it is chosen by information criteria.

A short run relationship equation is given by VECM (2, 2):

$$\begin{aligned}\Delta LCPI = \quad & -0.54 + 0.39\Delta LCPI(-1) + 0.26\Delta LCPI(-2) + 0.26\Delta LM3(-1) + 0.0004\Delta LM3(-2) \\ & + 0.046\Delta LNER(-1) - 0.0043\Delta LNER(-2) - 0.28\Delta LGDP(-1) + 0.0062\Delta LGDP(-2) \\ & + 0.005\Delta LTBR(-1) - 0.0013\Delta LTBR(-2) + 0.56\end{aligned}$$

For long run relationship C (1) should be negative and statistically significant. The results from the above equation showed that the error correction term is - 0.54 and significant at 5% level of significance this indicated that 5.4% is the speed of adjustment to restore equilibrium condition in the long run, this implies that in each quarter 5.4% of the errors will be corrected.

In short run, the results showed that lagged CPI are positive and highly significant at 5% level of significance; this implies that in short run inflation is caused by its previous values .The money supply is found to have short run relationship with inflation as its lagged values are positive and statistically significant at 5% level of significance whereas Exchange Rate, Gross Domestic Product and Treasury bills rate are not significant , we can say that they have no short run relationship with inflation.

In model checking, statistical test was applied to determine the adequacy of selected model, these are Breusch –Godfrey serial correlation test to detect if there is autocorrelation in residuals, Jarque –Bera test for normality and Breusch-Pagan-Godfrey for heteroskedasticity (Doornik, 1996) the result as shown in the TABLE VI below indicate that there is no serial correlation, no heteroskedasticity in residuals and there are normally distributed.

**TABLE VI. Diagnostic checking for VAR Model**

| Test | Statistic | Prob |
|---|---|---|
| Breusch-Godfrey-Correlation LM test | 1.307 | 0.27 |
| Jarque-Bera test | 0.609 | 0.73 |
| Breusch-Pagan-Godfrey | 0.698 | 0.77 |

As the model pass for diagnostic checking it can be served to emphasize the relationship between variables, Granger causality test (Lin, 2008) on VECM system is conducted and it has been found that changes in Monetary aggregate would predict changes in inflation and there is feedback relationship where inflation is also a good predictor of monetary aggregate; Money aggregate would predict exchange rate but exchange rate does not predict M3; CPI seems to cause GDP but GDP cannot causes CPI. Lastly change in exchange rate and treasury bills cannot cause change in inflation therefore they are not good predictor of inflation.

The impulse response function and the variance decomposition is also conducted; Impulse response identify the responsiveness of the dependent variable in a VAR system when a shock of other variable in VAR system is put to the error term whereas the variance decomposition analysis of statistics indicate the percentage contribution of innovations in each of the variables in the system to the variance of the inflation (Gospodinov et al., 2013).

For impulse response function, when one standard deviation positive shock is given to CPI, there is slow increase in inflation. If one positive standard deviation is given to money supply CPI increase positively up to period 3 then became negative up to quarter 10.One standard deviation positive shock to NER, CPI is positive at period one and after this period it decrease negatively up to period 5 then it is negative constant up 10 periods. When one positive standard deviation applied to GDP, from quarter one up to 5 CPI increase from 5 to seven period it is decreasing and in last periods it is constant. Lastly one positive standard deviation given to TBR, there is a steady increase up to period 6 then constant.

For variance decomposition, In short run for instance in quarter three, the results indicated that the innovation to CPI can cause 85.70% variation of the fluctuation in CPI(own shock). A shock to Money aggregate account for 1.22 % variation of the fluctuation in CPI, the impulse to exchange rate can cause 2.35%, for GDP account for 8.25% and those for TBR account for 2.48% variation of the fluctuation in CPI. In long run for instance in 8 periods, CPI can contribute 70.22 % variation of fluctuation in CPI, The contribution to CPI on its own variation is reduced but still have a great impact considered to other variables. Shock to money aggregate and treasury bills rate are very small in long run, final shock to GDP and NER had increase in long run and their contribution to CPI variance are11.38% and 12.46%.

**2. Forecasting:**

In forecasting inflation rate in Rwanda based on selected ARIMA and VAR models, the forecast period is from 2014Q1 to 2015Q1 and as shown in TABLE VII below, that fitted observations from ARIMA model are closed to actual observations than that produced from VAR model.

**TABLE VII. Out of sample forecast from ARIMA and VAR model**

| FORECAST PERIOD | ACTUAL VALUES | ARIMA | VAR |
|---|---|---|---|
| 2014Q1 | 100.70 | 100.38 | 98.52 |
| 2014Q2 | 100.10 | 101.42 | 99.52 |
| 2014Q3 | 101.10 | 102.51 | 100.24 |
| 2014Q4 | 100.70 | 100.58 | 97.99 |
| 2015Q1 | 100.82 | 103.30 | 99.22 |

The performance evaluation in forecasting from these two models is assessed by comparing different measures which are MAPE, MAE, RMSE and Theil Unequality, the best model in forecasting is the one which has small values and the results from TABLE VIII below indicates that values produced by ARIMA (3, 1, 4) are smaller than those for VAR model so ARIMA model outperform better than VAR model in forecasting inflation in Rwanda.

**TABLEVIII. Comparison of ARIMA and VAR model in forecasting inflation**

| Model | RMSE | MAE | MAPE | Theil U |
|---|---|---|---|---|
| ARIMA(3,1,4) | 0.98 | 0.79 | 0.78 | 0.004 |
| VECM(2,2) | 1.81 | 1.58 | 1.57 | 0.009 |

## 5. CONCLUSION

The aim of time series analysis is to understanding the past values of a given variable and predict it in the future using statistical models, the purpose of this research was to test and distinguish which of ARIMA and VAR models perform better to forecast inflation rate in Rwanda. These models were developed and we applied them on time series data which are consumer price index, money aggregate, exchange rate, gross domestic product and treasury bills rate collected from 2000Q1 up to 2015Q1 where inflation measured as CPI is dependent variable while others are independent variables. Firstly we determined the time series properties of all variables and based on ADF unit root test, the study showed that all variables are stationary after first difference.

In building ARIMA model based on Box-Jenkins approach, data on Consumer Price Index from the period of 2000Q1 up to 2013Q4 were used and based on information criteria and residual white noise, ARIMA (3,1, 4) was chosen as the good model in univariate family and the study showed that ARIMA model is useful to predict inflation in Rwanda. In vector autoregressive model, inflation is explained by others economic variables; testing long and short run relationship between variables in VAR system using Johansen cointegration test was conducted and it has been found that relationship exist. These relationship differ from one variable to another, money aggregate and exchange rate influence positively inflation while gross domestic product and treasury bills rate influence negatively.

The forecast from both models suggest that ARIMA model perform better than VAR model in predicting inflation in Rwanda. We suggest that ARIMA model may be efficient in forecasting short term periods as it has proven by vector error correction model. However, an inflationary process must be set with other variables in order to asses which variables are more likely to affect inflation either in short run or in long run, therefore by comparing the forecast performance of these two model based on RMSE, MAE and MAPE we can conclude that ARIMA model is better than VAR model in forecasting inflation in Rwanda.

## REFERENCES

[1] Alnaa, S. E., & Abdul-Mumuni, A. (2005). Predicting Inflation in Ghana: A Comparison of Cointegration and Arima Models. *SSRN Electronic Journal*, (September), 0–33.

[2] Bash, E. (2015). Applied Time Series :Modelling and Forecasting(Vol. 1).

[3] Burke, S. P., & Hunter, J. (2005). Modelling non-stationary economic time series: A multivariate approach.

[4] Casella, G., Fienberg, S., & Olkin, I. (2006).Time Series Analysis and Its Application, *Springer Texts in Statistics Design* (Vol. 102).

[5] Doornik, B. Y. J. A. (1996). Testing Vector Error Autocorrelation and Testing Vector Autocorrelation and Heteroscedasticity in Dynamic Models, (September).

[6] Gathingi, V. W. (2014). Modeling Inflation in Kenya Using ARIMA and VAR Models, (July).

[7] Gerardi, K., & Li, W. (2010). National Bank of Rwanda, Economic Review, (May).

[8] Gospodinov, N., Marı, A., & Pesavento, E. (2013). Unit Roots , Cointegration ,and Pretesting in Var. *Advances in Econometrics*, *32*(2013), 81–115.

[9] Helmut, L. (1980). Econometric Analysis with Vector Autoregressive Models, (1980).

[10] Johansen, S. (1995).Likelihood-Based Inference in Cointegrated Vector Autoregressive Models, *Advanced Texts in Econometrics*.

[11] KAYISIRE P.(2014).Single Equation Models for Inflation Forecast in Rwanda.

[12] Kollo, T., & Von Rosen, D. (2005). Advanced Multivariate Statistics with Matrices. *Computer*.

[13] Koopman, S. J., & Commandeur, J. J. F. (1994). Introduction to Time Series Analysis and Time, 134–144.

[14] Lack, C. (2006). Swiss National Bank Economic Studies.

[15] Lin, J. (2008). Notes on Testing Causality.

[16] Mahdi, E. (2011). Diagnostic Checking, Time Series and Regression, (August).

[17] Makridakis, S., & Hibon, M. (2008). Research Professor of Decision Sciences and Information Systems at INSEAD, Boulevard de Constance, Fontaineblea.

[18] Mccullough, B. D. (2005). Diagnostic Checks in Time Series. *Technometrics* (Vol. 47).

[19] Meyler, A. (2015). Forecasting irish inflation using ARIMA models, (August).

[20] Musoni J . Rutayisire (2008). Liberalization , monetary policy and demand for money in Rwanda ( 1980-2005 ).

[21] Ocran, M. K. (2003). A modelling of ghana's inflationary expereince: 1960-2003 1, 1–26.

[22] Thomas, S. (2009). Forecasting and measuring forecast accuracy.

[23] Wei, W. W. S. (n.d.). Time Series Analysis Univariate and Multivariate Methods Second edition.

[24] Ufitinema,R (2014.) Analysis of Factors Affecting Inflation Rate in Rwanda.

[25] Zivot, E., & Wang, J. (2006). Vector autoregressive models for multivariate time series. *Modeling Financial Time Series with SPLUS*, 383–427.